U.G. 5th Semester Examination - 2020

MATHEMATICS

Course Code: BMTMGERT10

Course Title: Basic of Higher Mathematics-I

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) If $\alpha, \beta, \gamma, \delta$ be the roots of biquadratic equation $x^4+qx+r=0$, find $\sum \alpha\beta$.

b) Under which conditions the equation $ax^3+bx^2+cx+d=0$ is a reciprocal equation of first kind?

c) Find the remainder when x^6+x^3+1 is divided by $(x+1)$.

d) If $z=1+i$, find $amp\, z$.

e) Find the modulus of the complex number $1+\sin\alpha+i\cos\alpha$.

f) What is the number of solutions of the non-homogeneous system of linear equations $x+y-z=2$ and $2x+2y-2z=8$?

g) Express the complex number $z=-i$ in polar form.

h) What are the Eigen values of the diagonal matrix $D=diag(d_1, d_2, d_3)$?

i) Write down the n-th derivative of the function $\log(a+x)$ with respect to x.

j) Give an example of a homogeneous function in x, y of degree zero.

k) If $u=\tan^{-1}\left(\frac{y}{x}\right)$, find $\frac{\partial u}{\partial y}$.

l) Define the rank of a matrix $A=(a_{ij})_{n\times n}$.

m) Define divergence of a vector point function.

n) If $\varphi(x, y, z)=x^2+y^2+z^2$, find ∇f.

o) When a vector field is called solenoidal?

2. Answer any **five** questions: 2×5=10

a) Find the product of all values of $(1+i)^{\frac{2}{3}}$.

b) Express $\frac{(\cos\theta+i\sin\theta)^4}{(\sin\theta+i\cos\theta)^3}$ in the form $(A+iB)$.

c) Find the roots of the characteristic equation of the matrix $\begin{pmatrix} 0 & 1 \\ -4 & 0 \end{pmatrix}$.

d) Find the rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & -4 \\ 0 & 4 & 5 \end{pmatrix}$.

e) Verify Euler's Theorem for the homogeneous function $u = x^2 - 5xy$.

f) Evaluate the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$.

g) If $a = (t^2, t, -t^3)$, $b = (\sin t, -2\cos t, 0)$, find $\frac{d}{dt}(\text{a.b})$.

h) If $r = x^3 i + 3yz^2 j - zk$, find $(\nabla . r)$ at the origin.

3. Answer any **two** questions: 5×2=10

a) i) If α, β, γ be the roots of the cubic equation $x^3 + qx - r = 0$, find the cubic equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$.

ii) Solve the binomial equation $x^5 - 1 = 0$. 3+2

b) i) Express the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

ii) If A and B be two orthogonal matrices of the same order, prove that AB is also an orthogonal matrix of the same order. 3+2

c) i) If r be the distance of $P(x, y, z)$ from the origin and r be the position vector of P relative to the origin, show that $curl\ r = 0$.

ii) State *Generalized Mean Value Theorem*. 3+2

4. Answer any **one** question: 10×1=10

a) i) Solve the cubic equation $x^3 + 3x = 0$ by Cardano's Method.

ii) Find the values of $(1-i)^{\frac{1}{4}}$. 5+5

b) i) Solve the following system of linear equations by Cramer's Rule:

$$2x - y = 3,\ 3y - 2z = 5,\ x + y + z = 1$$

ii) Find the Eigen values and Eigen vectors of the matrix $\begin{pmatrix} 2 & 1 \\ 5 & -3 \end{pmatrix}$. 4+(2+4)

c) i) If $y = \tan^{-1} x$, show that

$$(1 + x^2) y_{n+1} + 2nx\, y_n + n(n-1)\, y_{n-1} = 0.$$

Also find the value of $(y_n)_0$.

ii) If $r = (\cos nt)i + (\sin nt)j$; n being a parameter, then show that $r \times \frac{dr}{dt} = nk$. (4+2)+4

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